

S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages.

Also S.IC.6

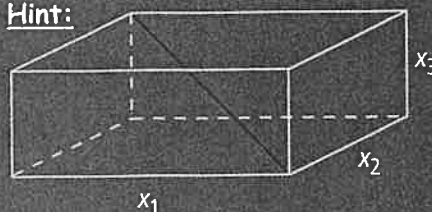
Objectives To find the standard deviation and variance of a set of values
To apply standard deviation and variance

SOLVE IT

Getting Ready!

For nonnegative x_i , how do
 $\sum_{i=1}^3 x_i$ and $\sqrt{\sum_{i=1}^3 (x_i)^2}$ compare?
Justify your answer.

Hint:



You learned about summation notation in Chapter 9. To find the mean of a data set, you sum the data values and divide by the number of data values. You can use summation to measure how data deviates from the mean.

Essential Understanding Standard deviation is a measure of how far the numbers in a data set deviate from the mean.

In the previous lesson you studied range and interquartile range. Each of these is a **measure of variation**. A measure of variation describes how the data in a data set are spread out.

Variance and **standard deviation** are measures showing how much data values deviate from the mean. The Greek letter σ (sigma) represents standard deviation. σ^2 (sigma squared) is the variance.

Take note

Key Concepts Finding Variance and Standard Deviation

- Find the mean, \bar{x} , of the n values in a data set.
- Find the difference, $x - \bar{x}$, between each value x and the mean.
- Square each difference, $(x - \bar{x})^2$.
- Find the average (mean) of these squares. This is the variance.

$$\sigma^2 = \frac{\sum (x - \bar{x})^2}{n}$$

- Take the square root of the variance. This is the standard deviation.

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

Problem 1 Finding Variance and Standard Deviation

What are the mean, variance, and standard deviation of these values?

6.9 8.7 7.6 4.8 9.0

$$\bar{x} = \frac{6.9 + 8.7 + 7.6 + 4.8 + 9.0}{5} = 7.4 \quad \text{Find the mean.}$$

Think

How can you organize your work? Use a table to record the values.

x	\bar{x}	$x - \bar{x}$	$(x - \bar{x})^2$
6.9	7.4	-0.5	0.25
8.7	7.4	1.3	1.69
7.6	7.4	0.2	0.04
4.8	7.4	-2.6	6.76
9.0	7.4	1.6	2.56
Sum			11.30

Make a table.

Find difference between each value and the mean. Square the differences.

Add the squares of the differences.

$$\sigma^2 = \frac{\sum(x - \bar{x})^2}{n} = \frac{11.3}{5} = 2.26 \quad \text{Find the variance.}$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{2.26} \approx 1.5 \quad \text{Find the standard deviation.}$$

The mean is 7.4. The variance is 2.26. The standard deviation is about 1.5.

Got It? 1. What are the mean, variance, and standard deviation of these values?
52 63 65 77 80 82

Problem 2 Using a Calculator to Find Standard Deviation STEM

Meteorology The table displays the number of U.S. hurricane strikes by decade from the years 1851 to 2000. What are the mean and standard deviation for this data set?

Decade	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Strikes	19	15	20	22	21	18	21	13	19	24	17	14	12	15	14

SOURCE: National Hurricane Center

Think

How do you know you are entering all the data values? The calculator value for n should match the number of table values.

Step 1 Use **STAT EDIT** to enter the data in list L1.

Step 2 In **STAT CALC** select the **1-Var Stats** option.

The mean is 17.6; the standard deviation is about 3.5.

1-Var Stats	
$\bar{x} = 17.6$	mean
$\Sigma x = 264$	
$\Sigma x^2 = 4832$	
$Sx = 3.641035959$	standard deviation
$\sigma x = 3.517574922$	
$n = 15$	

- Got It? 2. Meteorology** The table displays the number of hurricanes in the Atlantic Ocean from 1992 to 2006. What are the mean and standard deviation?

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number	4	4	3	11	10	3	10	8	8	9	4	7	9	14	5

Source: National Hurricane Center

In a data list, every value falls within some number of standard deviations of the mean. For example, if the mean is 50 and the standard deviation is 10, then a value x with $40 \leq x \leq 60$ is within one standard deviation of the mean.

Problem 3 Using Standard Deviation to Describe Data **STEM**

Meteorology Use the U.S. hurricane-strike data from Problem 2. Within how many standard deviations from the mean do all of the values fall?

Know

The data values, their mean, and their standard deviation

Need

The number of standard deviations from the mean that include all the data

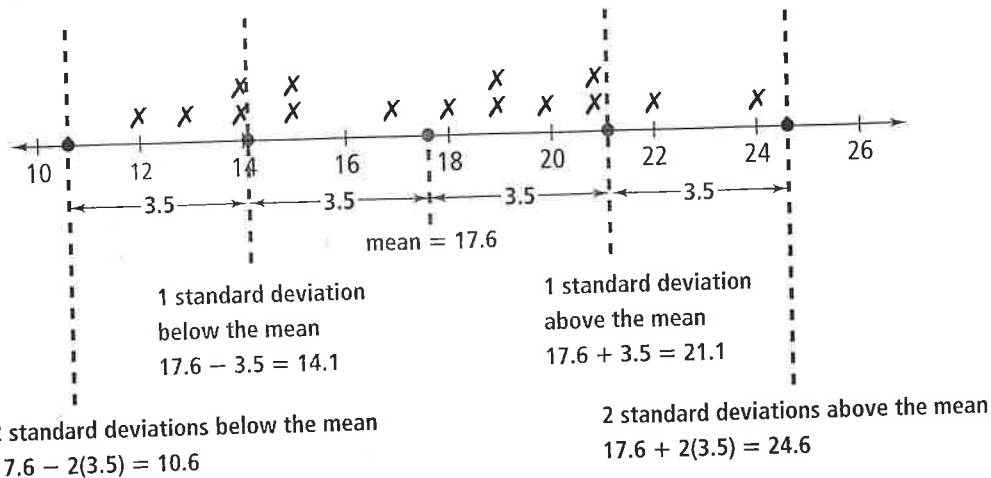
Plan

- Draw a number line.
- Plot the data values and the mean.
- Mark off intervals of 3.5 on either side of the mean.

Think

What is a good way to tell which values lie within each σ interval?

Plotting the values on a number line makes it easy to see the σ intervals.



All of the values fall within two standard deviations of the mean. Hurricane watchers can expect that the number of U.S. hurricane strikes in a decade will probably fall within two standard deviations of the 15-decade mean.

- Got It? 3. Meteorology** Use the Atlantic Ocean hurricane data from Got It 2.
- Within how many standard deviations of the mean do all of the values fall?
 - Reasoning** How might the U.S. Federal Emergency Management Agency (FEMA) use this information?



Lesson Check

Do you know HOW?

- Find the mean, variance, and standard deviation for the data set.
5, 15, 9, 3, 12, 8, 13, 6, 18, 11
- Within how many standard deviations of the mean do all of the data values fall?
12, 17, 15, 13, 9, 10, 12, 10, 15, 17

Do you UNDERSTAND?



- Vocabulary** Explain the difference between *measures of central tendency* and *measures of variation*.
- Compare and Contrast** Three data sets each have a mean of 70. Set A has a standard deviation of 10. Set B has a standard deviation of 5. Set C has a standard deviation of 20. Compare and contrast these 3 sets.
- Reasoning** What is the effect of an outlier on the standard deviation of a data set?



Practice and Problem-Solving Exercises



Practice

Find the mean, variance, and standard deviation for each data set.

◀ See Problem 1.

6. 78 90 456 673 111 381 21

7. 13 15 17 18 12 21 10

8. 12 3 2 4 5 7

9. 60 40 35 45 39



Graphing Calculator Find the mean and the standard deviation.

◀ See Problem 2.

10. The Dow Jones Industrial average for the first 12 weeks of 1988:

1911.31	1956.07	1903.51	1958.22	1910.48	1983.26
2014.59	2023.21	2057.86	2034.98	2087.37	2067.14

11. The Dow Jones Industrial average for the first 12 weeks of 2008:

12800.18	12606.30	12099.3	12207.17	12743.19	12182.13
12348.21	12381.02	12266.39	11893.69	11951.09	11972.25

Determine the whole number of standard deviations from the mean that include all data values.

◀ See Problem 3.

12. The mean price of the nonfiction books on a best-sellers list is \$25.07; the standard deviation is \$2.62.

\$26.95, \$22.95, \$24.00, \$24.95, \$29.95, \$19.95, \$24.95, \$24.00, \$27.95, \$25.00

13. The mean length of Beethoven's nine symphonies is 37 minutes; the standard deviation is 12 minutes.

27 min, 30 min, 47 min, 35 min, 30 min, 40 min, 35 min, 22 min, 65 min

14. Find the standard deviation for each data set. Use the standard deviations to compare each pair of data sets.
 fastest recorded speeds of various large wild cats (miles per hour):
 70 50 30 40 35 30 30 40 15
 fastest recorded speeds of various birds in flight (miles per hour):
 217 106 95 56 65 37 50 31 53 25 25 25

© 15. **Think About a Plan** Use the data for daily energy usage of a small town during ten days in June. Find the mean and the standard deviation of the data. How many values in the data set fall within one standard deviation from the mean? Within two standard deviations? Within three standard deviations?

51.8 MWh 53.6 MWh 54.7 MWh 51.9 MWh 49.3 MWh
 52.0 MWh 53.5 MWh 51.2 MWh 60.7 MWh 59.3 MWh

- How is the mean of the data set used in the formula for standard deviation?
- How can a table help you find the standard deviation?
- How can a graph help you decide how many standard deviations a data value is from the mean?

Income Use the chart at the right for Exercises 16–18.

16. Find the mean income for each year.

© 17. **Writing** Use the standard deviation for each year to describe how farm income varied from 2001 to 2002.

18. For 2001, the farm incomes of which states are not within one standard deviation of the mean?

19. a. **Data Collection** Make a table showing the number of siblings of each student in your class.

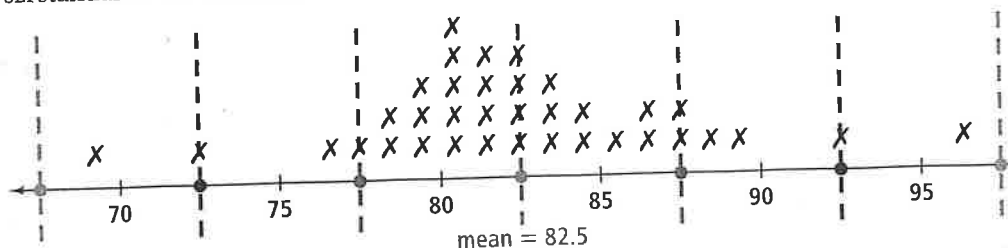
b. Find the mean and standard deviation of the data.

20. **Energy** The data for daily energy usage of a small town during ten days in January is shown.

83.8 MWh 87.1 MWh 92.5 MWh 80.6 MWh 82.4 MWh
 77.6 MWh 78.9 MWh 78.2 MWh 81.8 MWh 80.1 MWh

- a. Find the mean and the standard deviation of the data.
- b. How many values in the data set fall within one standard deviation from the mean? Within two standard deviations? Within three standard deviations?

© 21. **Error Analysis** One of your friends says that the data below fall within three standard deviations from the mean. Your other friend disagrees, saying that the data fall within six standard deviations from the mean. With whom do you agree? Explain.



Farm Income in Midwestern States (millions of dollars)

State	2001	2002
Iowa	10,653	10,834
Kansas	7979	7862
Minnesota	7537	7478
Missouri	4723	4402
Nebraska	9221	9589
North Dakota	2938	3223
South Dakota	3897	3779

SOURCE: U.S. Department of Agriculture

Challenge

22. a. Use the table to find the range, the mean, and the standard deviation of the ages for each team.
 b. **Reasoning** For two data sets, does the set with the greater range necessarily have the greater standard deviation? Support your answer with your results from part (a).
23. a. What effect will adding 10 to every value in a data set have on the standard deviation?
 b. What will be the effect if you multiply each value by 10?

Ages of the Members of Soccer Teams in an Adult League

Men	19	18	21	22	22	23
	23	23	23	22	36	22
	24	21	21	21	21	22
Women	26	31	22	24	20	22
	20	30	25	26	32	30
	23	33	21	27	24	28

Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

For Exercises 24–25, use the following bowling scores for six members of a bowling team: 175, 210, 180, 195, 208, 196.

24. What is the mean of the scores?
 25. What is the standard deviation of the scores?
 26. The 30th term of a finite arithmetic series is 4.4. The sum of the first 30 terms is 78. What is the value of the first term of the series?
 27. What is the probability of NOT getting a five when rolling a number cube? Write your answer as a fraction reduced to lowest terms.

Mixed Review

Make a box-and-whisker plot for each set of values.

28. 25, 25, 30, 35, 45, 45, 50, 55, 60, 60

29. 20, 23, 25, 36, 37, 38, 39, 50, 52, 55

Find the center and the radius of each circle.

30. $(x + 2)^2 + (y + 1)^2 = 36$

31. $(x - 1)^2 + (y - 1)^2 = 4$

Get Ready! To prepare for Lesson 11-8, do Exercises 32–37.

Simplify each radical expression.

32. $\frac{1}{\sqrt{4}}$

33. $-\frac{1}{\sqrt{9}}$

34. $\frac{1}{\sqrt{36}}$

35. $-\frac{1}{\sqrt{121}}$

36. $-\frac{1}{\sqrt{81}}$

37. $\frac{1}{\sqrt{49}}$

See Lesson 11-6.

See Lesson 10-3.

See Lesson 6-1.