11-5 Probability Models

Content Standards

S.MD.6 Students will use probabilities to make fair decisions . . .

S.MD.7 Students will analyze decisions using probability concepts . . .

Objective To use probabilities to make fair decisions and analyze decisions

Develop a plan for investigating this situation.

Solve It! Getting Ready!

Suppose you and your friends are trying to decide who gets to sit in the front seat of the car for a road trip. What are some ways that you and your friends can reach a fair decision? (Assume that none of you are driving.)

MATHEMATICAL PRACTICES

In the Solve It, you may have thought about ways that you have reached fair decisions. In this lesson, you will learn how to use probabilities to make fair decisions.

Lesson Vocabulary
- probability model

Essential Understanding You can use probability models to analyze situations and make fair decisions.

A fair game is a game in which all of the participants have an equal chance of winning. For example, two players take turns rolling a number cube. The first player gets a point if an odd number is rolled, and the second player gets a point if an even number is rolled. Each player has the same probability of scoring a point on each roll. Similarly, a fair decision is based on choices that are equally likely to be chosen.

Problem 1 Making a Fair Decision

A class of 25 students wants to choose 3 students at random to bring food for a class party. Any set of 3 students should have an equal chance of being chosen. Which of the following strategies will result in a fair decision?

Think
What is required for a decision to be fair?
Each possible outcome must be equally likely.

A **The students line up alphabetically, and each one in succession flips a fair coin. The first three students to flip heads bring the food.**

This strategy will not result in a fair decision because all students do not have an equal chance of being chosen. Students at the back of the line will probably never get to flip a coin.

B **Each student draws a card from a well-shuffled deck of 52 cards. The teacher shuffles a second deck of cards, spreads them out, and draws cards one by one until he matches the cards of three of the students.**

Assuming a good shuffle, this strategy gives any group of three students an equal chance of being selected. So this strategy will result in a fair decision.

Got It? **1.** Two siblings are trying to decide who has to mow the lawn this weekend. They decide to race, and the winner does not have to mow the lawn. Is the result a fair decision? Explain.

For an event to be random, there is no predetermined pattern or bias toward any particular outcome. You can use random numbers to help you make fair decisions.

A random number table contains randomly-generated digits from 0 to 9. You can use these numbers to model randomness for sampling purposes. Part of a random number table is shown below. Typically, space is inserted between groups of 5 numbers to help with readability.

Random Number						
87494	39707	20525	95704	48361	27556	34599
14164	15888	24997	82392	08525	47551	37304
61249	08241	16243	18371	03349	91759	53613
67868	56747	73521	05975	40411	49493	70904

Problem 2 Using Random Numbers

A coach wants to select 3 of her 15 basketball players at random to lead warm-ups before practice each week. The coach assigns each player a number from 01 to 15. How can the coach use the random number table to fairly choose the three players?

Step 1 Choose a line of digits from a random number table.

87494 39707 20525 95704 48361 27556 34599

Step 2 Read the digits from the random number table in consecutive pairs until you get the numbers for three of the players. Skip numbers like 87 and 49 because they do not represent any of the 15 players. Also ignore any duplicates that may occur.

87494 39707 20525 95704 48361 27556 34599

The coach chooses the players assigned numbers 07, 04, and 12 to lead warm-ups.

Plan

How do you handle the spaces in the digits?
Ignore the spaces between the digits. For example, the third two-digit number is 43.

Got It? **2. a.** A teacher wants to organize 10 students into two teams for a math game. The teacher assigns each student a number from 0 to 9. He uses the second line of digits from the random number table above to select the teams. He alternates the assigned team as each student is chosen. Which numbers will be used to create team 1?

b. Reasoning Is this a fair way to determine the teams? Explain.

You can use a **probability model** to assign probabilities to outcomes of a chance process. In Lesson 11-2, you learned that you can use a simulation to estimate the experimental probability of an event. A simulation is an example of a probability model. You can use probability and simulations to make predictions about real-life situations.

Problem 3 Modeling with a Simulation

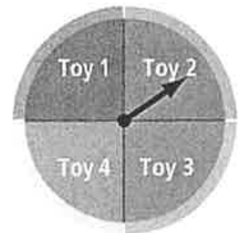
A restaurant gives away 4 different toys in their kids' meals. Each meal contains exactly one toy, and the toys are equally and randomly distributed. About how many kids' meals would a parent expect to have to buy to get all 4 toys?

Know
You know that toys are randomly distributed in each kids' meal. There is 1 toy in each meal.

Need
A probability model that generates four equally-likely events

Plan
You can use a spinner with four equal sections to simulate the random choosing of each toy.

Step 1 Spin the spinner and keep track of the results in a frequency table. Continue spinning until you have simulated getting each toy at least once.



Kids' Meal Simulation

Toy	Tally	Frequency
1	//	2
2	///	3
3	/	1
4	//	2

The results of this trial indicate that you would expect to have to buy $2 + 3 + 1 + 2 = 8$ kids' meals in order to get all 4 different toys.

Think
Why do you need to repeat the simulation?
In general, the number of kids' meals you have to buy to collect all four toys will vary.

Step 2 Repeat the simulation several times.
Suppose that you conduct the simulation 24 more times and the results indicate that you can expect to have to buy the meal the following number of times before you get all 4 toys.

7, 9, 12, 6, 15, 5, 8, 10, 7, 13, 11, 6, 8, 9, 12, 13, 9, 10, 12, 7, 11, 5, 6, and 8.

Step 3 Find the average number of kids' meals needed to get all 4 toys.

$\frac{227}{25} = 9.08$

On average, you will need to buy about 9 kids' meals in order to get all 4 toys.

Got It? **3.** Suppose that you are playing a board game for which you must roll a 6 on a number cube before you are able to move your game piece from start. Describe a simulation you can use to predict the number of times you would expect to have to roll the number cube before you can move from start.

You can use contingency tables to determine conditional probabilities and use those probabilities to evaluate decisions.

Problem 4 Using Probability to Analyze Decisions

A pharmaceutical company is testing the effectiveness of a new drug for asthma patients. Out of 160 volunteers who suffer from asthma, 80 are given the drug, and 80 are given a placebo, which has no active ingredients. After 2 weeks, the volunteers are asked if they noticed improvement in their asthma symptoms. The results of the survey are shown in the contingency table at the right.

	Improved	Did not Improve	Totals
Received the Drug	67	13	80
Received the Placebo	24	56	80
Totals	91	69	160

A What is the probability that a volunteer reported noticeable improvement in symptoms given that he received the test drug?

The first row represents the total number of volunteers who received the drug. The number of those volunteers that reported improvements in symptoms is 67.

$$P(\text{improvement} \mid \text{drug}) = \frac{\text{number of volunteers who improved}}{\text{number of volunteers who received the drug}} = \frac{67}{80} = 0.8375$$

B What is the probability that a volunteer received the placebo given that he did not report a noticeable improvement in symptoms?

The second column of data shows the total number of volunteers who did not report improvements in symptoms, 69. The second row represents the total number of volunteers who received a placebo.

$$P(\text{placebo} \mid \text{no improvement}) = \frac{56}{69} \approx 0.8116$$

Think

How can you use what you know to analyze the company's decision?
You can use the probabilities to analyze whether the drug was actually effective.

C The pharmaceutical company decides to produce and distribute this drug. The drug is marketed as an effective way to improve the symptoms of asthma. Based on the results of the test, did the company make a good decision? Explain.

The probabilities calculated above show that about 8 out of 10 people who are given the drug experience improvements in symptoms. Also, about 7 out of 10 who receive the placebo do not experience improvement in symptoms. Based on this study, the new drug appears to be effective at treating asthma symptoms. The company made a good decision.

Got It? 4. The results of another test are shown in the contingency table below. Should the company produce and distribute the new drug? Explain.

	Reported Improvements	Did not Report Improvements	Totals
Received the Drug	23	27	50
Received the Placebo	19	31	50
Totals	42	58	100

Lesson Check

Do you know HOW?

Use the contingency table for Exercises 1 and 2.

	Passed	Failed	Totals
Studied	8	1	9
Did not Study	3	6	9
Totals	11	7	18

1. Find $P(\text{passed} \mid \text{studied})$.
2. Find $P(\text{did not study} \mid \text{failed})$.

Do you UNDERSTAND? MATHEMATICAL PRACTICES

3. **Open Ended** Give an example of a fair decision and an example of an unfair decision if two brothers are trying to decide who has to wash the dishes.
4. **Error Analysis** A classmate conducted a simulation to predict how many boxes of cereal he would need to buy to get all 5 prizes. After one trial of the simulation, he concluded that he would need to buy 7 boxes of cereal. What was your classmate's error? How should he conduct the simulation?
5. **Vocabulary** Look up the definition of simulation in a dictionary or online. How does this definition relate to the concept of simulation in mathematics?

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

For Exercises 6–7, determine whether strategies described result in a fair decision. Explain. See Problem 1.

6. There are 24 students in math class. The teacher wants to choose 4 students at random to come to the board and work a math problem. She writes each students name on a slip of paper, places them in a hat, and chooses 4 without looking.
7. You and three friends want to choose which two of your group will shovel the snow in the driveway. You are each assigned a number from 1 to 4, and then a spinner to choose the first person. Then that person chooses the second person.

For Exercises 8–9, use the lines from a random number table below. See Problem 2.

84496 18732 60330 19536 58380 52544 48712
01603 48862 18519 29834 90890 69751 20514

8. The advisor of the Good Citizen's club wants to select 4 of its 25 members to raise and lower the flag each day this week. She assigns a two-digit number from 01 to 25 to each student. What are the numbers that correspond to the members who will raise and lower the flag?
9. **Camp** A coach wants to select 5 of his 16 players at random to help with a youth basketball camp this weekend. He assigns a two-digit number from 01 to 16 to each player. What are the numbers of the players who will help at the camp?
10. **Shopping** A grocery store is giving away scratch-off tickets to each customer when they spend over \$50. There are 3 different discount offers randomly and equally distributed among the scratch-off tickets. What is a simulation you could use to find, on average, how many times a customer would have to spend over \$50 in order to get all 3 different discount offers? See Problem 3.

11. **Testing** The contingency table below shows the number of nursing students who took preparatory class before taking their board exams and the number of students who passed the board exams on their first attempt.

See Problem 4.

	Preparatory Class	No Preparatory Class	Totals
Passed Exams	14	11	25
Did not Pass Exams	3	6	9
Totals	17	17	34

a. What is the probability that a nursing student passed the board exams given that he or she took the preparatory class?
b. What is the probability that a nursing student did not pass the board exams given that he or she did not take the preparatory class?
c. A student decides to take the preparatory class before he takes the board exams. Is this a good decision? Explain.

B Apply

12. **Community Service** There are 24 members in a school's drama club. The advisor wants to randomly select 8 members to help seat patrons prior to a play at a local theater. How can the advisor choose the 8 members fairly? Explain.

Suppose you take a multiple-choice quiz. There are 4 choices for each question. You do not know the answers to the last 5 questions, so you guess the answers. For Exercises 13–14, use the simulation results below, where 1 represents a correct answer and 2, 3, and 4 represent incorrect answers.

31242 41211 34141 41212 23342 23242 13412
11313 42433 32334 31234 13314 41432 23413
42322 14331 12224 12232 31232 22223 31233
11214 33243 33414 21224 34112 43432 14234

13. How many trials of the simulation were conducted? What was the average number of correct answers for these trials?

14. If you need at least 3 correct answers to earn a passing grade, what is the experimental probability of guessing the answers and getting a passing grade based on this simulation?

C 15. **Think About a Plan** A delivery company is evaluating the effectiveness of a defensive driving course. The contingency table at the right displays data about drivers who took the course. Based on these results, the company decides to continue to offer the defensive driving course. Is this a good decision? Explain.

	Took Course	Did not Take Course	Totals
No Major Accidents	3	18	21
At least 1 Major Accident	0	4	4
Totals	3	22	25

- What probabilities do you need to analyze the decision?
- How do you decide whether the course is effective?

16. **Writing** What role does probability play in decision-making and problem solving? Support your answer with examples.

17. **Sports** The local football team's field goal kicker has made 16 out of 20 field goal attempts from less than 30 yards so far this season. So, the experimental probability that the kicker will make his next field goal kick is 80%. What simulation could you use to find the average number of field goals he will make if he has three attempts of less than 30 yards in a game?

Challenge

18. In some contests, the prizes are randomly distributed, but there may be more of one kind of prize than another. Suppose there are 250 tickets in a raffle. There is 1 grand prize, 5 first prizes, and 20 second prizes available. How can you simulate the results of the raffle?

Standardized Test Prep

SAT/ACT

19. Four customers arrive at a store at the same time to be the first to buy the new release of a video game. Since there are only three copies left, the store manager assigns each customer a number from 1 to 4, and uses the random numbers below to choose which customers get to buy the game. Which customer does NOT get to buy the game?

92352 46423 10770 44286 17178 25060 74858

(A) customer 1 (B) customer 2 (C) customer 3 (D) customer 4

20. Which explicit formula represents the geometric sequence 5, 15, 45, 135, . . . ?

(F) $a_n = 5(3)^{n-1}$ (G) $a_n = 3(5)^{n-1}$ (H) $a_n = 5^{n-1}$ (I) $a_n = 5(3)^n$

21. What is the completely factored form of the expression $3x^2 - 9x - 12$?

(A) $(x - 1)(x + 4)$ (C) $3(x - 4)(x + 1)$
(B) $-3(x + 1)(x - 4)$ (D) $(3x + 3)(x - 4)$

Short Response

22. What is the simplest form of the expression $5\sqrt{50x^5y^3} \cdot 2\sqrt{48xy}$?

Mixed Review

***D* and *F* are independent events. $P(D) = 0.35$ and $P(F) = 0.52$.**

See Lesson 11-4.

23. What is $P(D \mid F)$?

24. What is $P(F \mid D)$?

Get Ready! **To prepare for Lesson 11-6, do Exercises 25–26.**

See p. 983

Find the median of each data set.

25. 0.2 0.3 0.6 1.2 0.7 0.9 0.8

26. 11 23 15 17 21 18 21

Mid-Chapter Quiz

MathXL for School
Go to PowerAlgebra.com

Do you know HOW?

Evaluate each expression.

1. $4!$

2. $6!$

3. $\frac{5!}{3!}$

4. $\frac{6!}{4!2!}$

5. ${}_7C_3$

6. ${}_9C_8$

7. ${}_5P_2$

8. ${}_{11}P_9$

9. ${}_4C_4$

10. ${}_4P_4$

11. $2({}_5C_4) - {}_3C_2$

12. $3({}_3P_2) + {}_3P_1$

Indicate whether each situation involves a combination or permutation. Then solve.

13. How many ways are there to select five actors from a troupe of nine to improvise a scene?

14. How many different three-student study groups can be formed from a class of 15?

15. Your teacher is looking for a new apartment. There are five apartments available. In how many ways can your teacher inspect the apartments?

Suppose you select a number at random from the sample space {5, 6, 7, 8, 9, 10, 11, 12, 13, 14}. Find each probability.

16. $P(7)$

17. $P(5 \text{ or } 13)$

18. $P(\text{greater than } 10)$

19. $P(\text{multiple of } 30)$

20. $P(\text{less than } 7 \text{ or greater than } 10)$

21. $P(\text{greater than } 6 \text{ and less than } 12)$

22. $P(\text{integer})$

23. $P(\text{less than } 10 \mid \text{less than } 13)$

24. $P(\text{greater than } 8 \mid \text{less than } 11)$

25. $P(\text{greater than } 7 \mid \text{greater than } 12)$

Two standard number cubes are tossed. State whether the events are mutually exclusive. Then find $P(A \text{ or } B)$.

26. A means their sum is 12; B means both are odd.

27. A means they are equal; B means their sum is a multiple of 3.

Do you UNDERSTAND?

28. Vocabulary Explain the difference between experimental probability and theoretical probability.

29. Suppose you select a number at random from the set {90, 91, 92, . . . , 99}. Event A is selecting a multiple of 2. Event B is selecting a multiple of 3.

a. Writing Are events A and B mutually exclusive? Are they independent? Explain your answers.

b. Find $P(A)$ and $P(B)$.

c. Find $P(A \text{ and } B)$.

d. Find $P(A \text{ or } B)$.

e. Find $P(A \mid B)$ and $P(B \mid A)$.

30. Reasoning Let F and G be mutually exclusive events. Event F occurs more frequently than event G. Write the following in order from least to greatest: $P(F)$, $P(G)$, $P(F \text{ or } G)$, $P(G \mid F)$.

31. Error Analysis For two events A and B, a student calculates the probabilities shown. Explain how you can tell that the student made a mistake.

~~P(A and B) = 0.35~~
~~P(A | B) = 0.29~~

32. Writing A local restaurant owner employs 6 high school students who all want to work the same shift during spring break vacation week. To choose which 2 students will can work the shift, the owner assigns each student employee a number between 1 and 6, and then she rolls a standard number cube twice. The numbers that the number cubes show represent the employees who can work the shift. (If there are doubles, she rolls again.) Is the result a fair decision? Explain.