

Objectives To find conditional probabilities
To use tables and tree diagrams to determine conditional probabilities



This sounds like a great idea for a television game show!



Getting Ready!

A great prize is inside Box 1, 2, or 3. You choose Box 1. You are shown that Box 2 contains a rubber chicken. You are given a chance to change your choice. Should you stay with Box 1 or should you change to Box 3? Justify your answer.



MATHEMATICAL PRACTICES

The probability that an event, B , will occur given that another event, A , has already occurred is called a **conditional probability**.



Lesson Vocabulary

- conditional probability
- contingency table

Essential Understanding Conditional probability exists when two events are dependent.

You write the conditional probability of event B , given that event A occurs, as $P(B | A)$. You read $P(B | A)$ as “the probability of event B , given event A .”

A **contingency table**, or *two-way frequency table*, is a frequency table that contains data from two different categories. Contingency tables and tree diagrams can help you find conditional probabilities.



Problem 1 Finding Conditional Probability

Education The table shows students by gender and by type of school in 2005. You pick a student at random.

A What is $P(\text{female} | \text{graduate school})$?

The condition that the person selected is at graduate school limits the sample space to the 3,303,000 graduate students. Of those, 1,954,000 are female.

$$P(\text{female} | \text{graduate school}) = \frac{1954}{3303} \approx 0.59$$

Student Genders

	Males (in thousands)	Females (in thousands)
Two-year colleges	1866	2462
Four-year colleges	4324	5517
Graduate schools	1349	1954

SOURCE: U.S. Census Bureau

Think

What's the condition?
The student is at a graduate school.

B What is $P(\text{female})$?

$$P(F) = \frac{\text{total number of females}}{\text{total number of students}} = \frac{2462 + 5517 + 1954}{1866 + 2462 + 4324 + 5517 + 1349 + 1954}$$
$$= \frac{9933}{17,472} \approx 0.57$$

- Got It?** 1. a. In Problem 1, what is $P(\text{Four-year} \mid \text{male})$?
- b. **Reasoning** Without calculating, given a student is enrolled in a four-year college, is it more likely for the student to be male or female? Explain.

Problem 2 Conditional Probability in Statistics

Multiple Choice Americans recycle increasing amounts through municipal waste collection. The table shows the collection data for 2007. What is the probability that a sample of recycled waste is paper?

- (A) 16% (C) 33%
- (B) 28% (D) 57%

The given condition is that the waste is *recycled*. A favorable outcome is that the recycled waste is paper.

$$P(\text{paper} \mid \text{recycled}) = \frac{45.2}{45.2 + 7.2 + 3.2 + 2.1 + 21.7}$$
$$\approx 0.57, \text{ or } 57\%$$

The probability that the recycled waste is paper is about 57%. The correct answer is D.

- Got It?** 2. a. What is the probability that a sample of recycled waste is plastic?
- b. What is the probability that a sample of recycled waste is glass?

Municipal Waste Collected
(millions of tons)

Material	Recycled	Not Recycled
Paper	45.2	37.8
Metal	7.2	13.6
Glass	3.2	10.4
Plastic	2.1	28.6
Other	21.7	46.3

SOURCE: U.S. Environmental Protection Agency

Think
What's the condition?
The waste sample has to be recycled waste.

You can use a formula to find conditional probability.

Take note

Key Concept Conditional Probability

For any two events A and B with $P(A) \neq 0$,

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

Using the formula for conditional probability, you can calculate a conditional probability from other probabilities.

Problem 3 Using the Conditional Probability Formula

Market Research A utility company asked 50 of its customers whether they pay their bills online or by mail. What is the probability that a customer pays the bill online, given that the customer is male?

Bill Payment

	Online	By Mail
Male	12	8
Female	24	6

Think

To use $P(\text{online} \mid \text{male})$ you need $P(\text{male and online})$ and $P(\text{male})$.

There are 20 males, and 12 males who pay online out of 50 total customers. Substitute and simplify.

Write

$$P(\text{male and online}) = \frac{12}{50}$$

$$P(\text{male}) = \frac{20}{50}$$

$$P(\text{online} \mid \text{male}) = \frac{P(\text{male and online})}{P(\text{male})}$$

$$= \frac{\frac{12}{50}}{\frac{20}{50}}$$

$$= \frac{12}{20} = \frac{3}{5} = 0.6$$

The probability that a customer pays online given that the customer is male is 0.6.

- Got It?** 3. Researchers asked shampoo users whether they apply shampoo directly to the head, or indirectly using a hand. What is the probability that a respondent applies shampoo directly to the head, given that the respondent is female?

Applying Shampoo

	Directly Onto Head	Into Hand First
Male	2	18
Female	6	24

It follows from $P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$ that $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$.

You can use this rule along with a tree diagram to find probabilities of dependent events.

Problem 4 Using a Tree Diagram

Education A school system compiled the following information from a survey it sent to people who were juniors 10 years earlier.

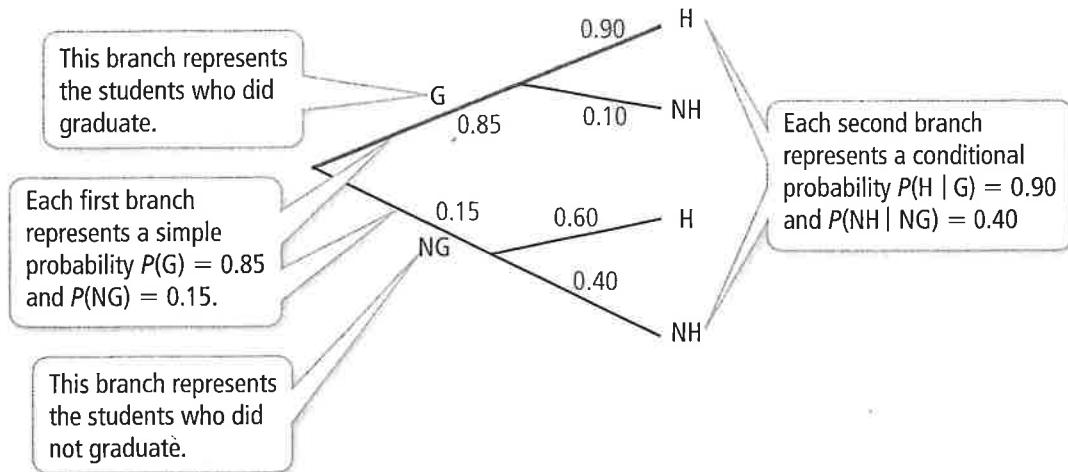
- 85% of the students graduated from high school.
- Of the students who graduated from high school, 90% are happy with their present jobs.
- Of the students who did not graduate from high school, 60% are happy with their present jobs.

What is the probability that a person from the junior class 10 years ago graduated from high school and is happy with his or her present job?

Make a tree diagram to help organize the information.

Let G = graduated, NG = not graduated, H = happy with present job, and NH = not happy with present job.

Think
What are the branches at each point?
The tree first branches at "graduated" and "not graduated." Each of these branches at "happy" and "not happy."



Think
Which path should you follow?
Follow the path that represents graduates who are happy with their present job.

The blue highlighted path represents $P(G \text{ and } H)$.

$$\begin{aligned} P(G \text{ and } H) &= P(G) \cdot P(H | G) \\ &= 0.85 \cdot 0.90 \\ &= 0.765 \end{aligned}$$

The probability that a person from the junior class 10 years ago graduated and is happy with his or her present job is 0.765, or 76.5%.

Got It? 4. What is the probability that a student from the junior class 10 years ago in Problem 4 did not graduate and is happy with his or her present job?



Lesson Check

Do you know HOW?

A card is drawn from a standard deck of cards. Find each probability, given that the card drawn is black.

- $P(\text{club})$
- $P(4)$
- $P(\text{diamond})$
- The probability that a car has two doors, given that it is red, is 0.6. The probability that a car has two doors *and* is red is 0.2. What is the probability that a car is red?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

- Reasoning** Using the tree diagram in Problem 4, explain why the probabilities on each pair of branches must add up to 1.
- Open-Ended** Describe a situation in which you would use conditional probability to find the answer.
- Compare and Contrast** How are the Fundamental Counting Principle and tree diagrams alike? How are they different?



Practice and Problem-Solving Exercises



Practice

Use the table to find each probability.

- $P(\text{has diploma})$
- $P(\text{has diploma and experience})$
- $P(\text{has experience} \mid \text{has diploma})$
- $P(\text{has no diploma} \mid \text{has experience})$

Use the table to find each probability.

- $P(\text{The recipient is male.})$
- $P(\text{The degree is a bachelor's.})$
- $P(\text{The recipient is female, given that the degree is an associate's.})$
- $P(\text{The degree is not an associate's, given that the recipient is male.})$

Use the survey results for Exercises 16 and 17.

- Find the probability that a respondent has a pet, given that the respondent has had a pet.
- Find the probability that a respondent has never had a pet, given that the respondent does not have a pet now.
- Sports** A football team has a 70% chance of winning when it doesn't snow, but only a 40% chance of winning when it snows. Suppose there is a 50% chance of snow. Make a tree diagram to find the probability that the team will win.

◀ See Problems 1 and 2.

Characteristics of Job Applicants

		Has Experience	
		Yes	No
Has High School Diploma	Yes	54	27
	No	5	4

Projected Number of Degree Recipients in 2010 (thousands)

Degree	Male	Female
Associate's	245	433
Bachelor's	598	858

SOURCE: U.S. National Center for Education Statistics

◀ See Problems 3 and 4.

39% have a pet now and have had a pet.
 61% do not have a pet now.
 86% have had a pet.
 14% do not have a pet now and have never had a pet.

19. Make a tree diagram based on the survey results below. Then find P (a female respondent is left-handed) and P (a respondent is both male and right-handed).
- Of all the respondents, 17% are male.
 - Of the male respondents, 33% are left-handed.
 - Of female respondents, 90% are right-handed.

B Apply

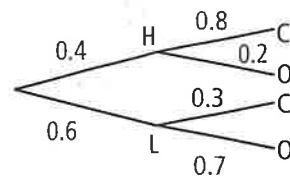
20. Suppose A and B are independent events, with $P(A) = 0.60$ and $P(B) = 0.25$. Find each probability.
- $P(A \text{ and } B)$
 - $P(A | B)$
- c. What do you notice about $P(A)$ and $P(A | B)$?
- d. **Reasoning** One way to describe A and B as independent events is *The occurrence of B has no effect on the probability of A .* Explain how the answer to part (c) illustrates this relationship.
21. **Think About a Plan** A math teacher gives her class two tests. 60% of the class passes both tests and 80% of the class passes the first test. What percent of those who pass the first test also pass the second test?
- What conditional probability are you looking for?
 - How can a tree diagram help you solve this problem?

Weather Use probability notation to describe the chance of each event. Let S , C , W , and R represent sunny, cloudy, windy, and rainy weather, respectively.

22. cloudy weather 23. sunny and windy weather 24. rainy weather if it is windy
25. **Transportation** You can take Bus 65 or Bus 79. You take the first bus that arrives. The probability that Bus 65 arrives first is 75%. There is a 40% chance that Bus 65 picks up passengers along the way. There is a 60% chance that Bus 79 picks up passengers. Your bus picked up passengers. What is the probability that it was Bus 65?

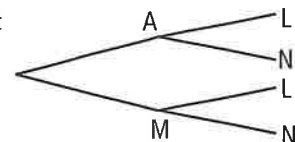
The tree diagram relates snowfall and school closings. Find each probability. Let H , L , O , and C represent heavy snowfall, light snowfall, schools open, and schools closed, respectively.

26. $P(C)$ 27. $P(H \text{ and } O)$ 28. $P(H | C)$
 29. $P(L | O)$ 30. $P(L | C)$ 31. $P(H | O)$



C Challenge

32. a. **Writing** Explain which branches of the tree diagram at the right represent conditional probabilities. Give a specific example.
 b. Are the event of having a license and the event of being an adult independent events? Justify your answer.
 c. **Open-Ended** Estimate probabilities for each branch of the tree diagram for your city or town. Then find $P(L)$.
33. **Reasoning** Sixty percent of a company's sales representatives have completed training seminars. Of these, 80% have had increased sales. Overall, 56% of the representatives (whether trained or not) have had increased sales. Use a tree diagram to find the probability of increased sales, given that a representative has not been trained.



A = adult (21 or older)
 M = minor (under 21)
 L = licensed driver
 N = not licensed to drive

Standardized Test Prep

Use the table for Exercises 34–35. A school library classifies its books as hardback or paperback, fiction or nonfiction, and illustrated or non-illustrated.

		Illustrated	Non-illustrated
Hardback	Fiction	420	780
	Nonfiction	590	250
Paperback	Fiction	150	430
	Nonfiction	110	880

SAT/ACT

34. What is the probability that a book selected at random is a paperback, given that it is illustrated?

(A) $\frac{260}{3610}$ (B) $\frac{150}{1270}$ (C) $\frac{260}{1270}$ (D) $\frac{110}{150}$

35. What is the probability that a book selected at random is nonfiction, given that it is a non-illustrated hardback?

(F) $\frac{250}{2040}$ (G) $\frac{780}{1030}$ (H) $\frac{250}{1030}$ (I) $\frac{250}{780}$

36. Which of the following expressions is equivalent to $3(n - 3)(n + 4)$?

(A) $3n^2 + 3n - 36$ (C) $3n^2 - 3n - 36$
 (B) $3n^2 - 3n + 36$ (D) $3n^2 - 36$

Short Response

37. What is the sample space for spinning the spinner twice? Are all the outcomes equally likely?



Mixed Review

Q and R are independent events. Find $P(Q \text{ and } R)$.

See Lesson 11-3.

38. $P(Q) = \frac{3}{4}$; $P(R) = \frac{4}{9}$

39. $P(Q) = \frac{17}{20}$; $P(R) = \frac{5}{19}$

Write an equation of a parabola with the given vertex and focus.

See Lesson 10-2.

40. vertex $(5, 2)$; focus $(6, 2)$

41. vertex $(-2, 3)$; focus $(-2, 6)$

Solve each equation. If necessary, round to the nearest thousandth.

See Lesson 7-5.

42. $2^x = 4$

43. $4^{2x} = 10$

44. $4^{x+1} = 28$

45. $7 - 3^x = -38$

Get Ready! To prepare for Lesson 11-5, do Exercises 46–49.

Find the following theoretical probabilities for the spinner at the right.

46. $P(\text{red})$

47. $P(\text{green})$

48. $P(\text{blue})$

49. $P(\text{yellow})$

