11-3 Probability of Multiple Events

Content Standards

S.CP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

Also S.CP.2, S.CP.5

Objectives To find the probability of the event A and B
To find the probability of the event A or B

Make sure you understand the game. What is the difference between the words "or" and "and"?

Getting Ready!

You and your friend take turns rolling two standard number cubes. If you roll a sum that is either odd or a prime number, you score a point. If your friend rolls a sum that is both odd and a prime number, she scores a point. Which score is likely yours? About how many turns have each of you taken? Explain.

MATHEMATICAL PRACTICES

You can find the probabilities of multiple events occurring by using the probabilities of the individual events.

Essential Understanding To find the probability of two events occurring together, you have to decide whether one event occurring affects the other event.

When the occurrence of one event affects how a second event can occur, the events are **dependent events**. Otherwise, the events are **independent events**.

Lesson Vocabulary
- dependent events
- independent events
- mutually exclusive events

Problem 1 Classifying Events

Are the outcomes of each trial dependent or independent events?

Think
What must you ask yourself?
Does the first event have any effect on the outcome of the second event?

A Roll a number cube. Then spin a spinner.

The two events do not affect each other. They are independent.

B Pick one flash card, then another from a stack of 30 flash cards.

Picking the first card affects the possible outcomes of picking the second card. The events are dependent.

Got It? **1.** You select a coin at random from your pocket. You replace the coin and select again. Are your selections independent events? Explain.

Multiply to find the probability that two independent events will both occur.

Take Note

Key Concept Probability of A and B

If A and B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

Dynamic Activity
Independent and Dependent Events

Problem 2 Finding the Probability of Independent Events

Picnic At a picnic there are 10 diet drinks and 5 regular drinks. There are also 8 bags of fat-free chips and 12 bags of regular chips. If you grab a drink and a bag of chips without looking, what is the probability that you get a diet drink and fat-free chips?

Think
Is it important that you don't look?
Yes; probability is based on random events. It is not random if you look.

Event A = picking a diet drink Event B = picking fat-free chips

A and B are independent. Picking a drink has no effect on picking the chips.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

$$= \frac{\text{number of diet drinks}}{\text{total number of drinks}} \cdot \frac{\text{number of bags of fat-free chips}}{\text{total number of bags of chips}}$$

$$= \frac{10}{15} \cdot \frac{8}{20} = \frac{4}{15} \approx 0.267, \text{ or } 26.7\%$$

The probability that you get a diet drink and fat-free chips is about 26.7%.

Got It? 2. In Problem 2, what is the probability that you get a regular drink and regular chips?

Two events that cannot happen at the same time are **mutually exclusive events**. If A and B are mutually exclusive events, then $P(A \text{ and } B) = 0$.

Problem 3 Mutually Exclusive Events

You roll a standard number cube. Are the events mutually exclusive? Explain.

Think
Can you roll a 2 and a 3 at the same time?
No; just one number comes up on one roll of one number cube.

A **rolling a 2 and a 3**

You cannot roll a 2 and 3 at the same time. The events are mutually exclusive.

B **rolling an even number and a multiple of 3**

You can roll a 6—an even number and a multiple of 3—at the same time. The events are not mutually exclusive.

Got It? 3. You roll a standard number cube. Are the events mutually exclusive? Explain.
a. rolling an even number and rolling a prime number
b. rolling an even number and rolling a number less than 2

To find the probability of either event A or event B occuring, you need to determine whether events A and B are mutually exclusive.

Take note

Key Concept Probability of A or B

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

If A and B are mutually exclusive events, then $P(A \text{ or } B) = P(A) + P(B)$.

Problem 4 Finding Probability for Mutually Exclusive Events

Languages At your high school, a student can take one foreign language each term. About 37% of the students take Spanish. About 15% of the students take French. What is the probability that a student chosen at random is taking Spanish or French?

Know
- The percentages of students taking Spanish or French.
- Students can take one foreign language at a time.

Need
The probability that a student is taking Spanish or French.

Plan
Use the correct formula for $P(A \text{ or } B)$.

"One foreign language each term" means the events are mutually exclusive.

$$\begin{aligned} P(\text{Spanish or French}) &= P(\text{Spanish}) + P(\text{French}) \\ &\approx 0.37 + 0.15 \\ &= 0.52 \end{aligned}$$

$P(A \text{ or } B) = P(A) + P(B)$ for mutually exclusive events.

The probability that a student chosen at random is taking Spanish or French is about 0.52, or about 52%.

Got It? **4. a.** In Problem 4, about 9% of the students take Mandarin Chinese. What is the probability that a student chosen at random is taking Spanish, French, or Mandarin Chinese?

b. Reasoning Without knowing the number of students in the school in Problem 4, can you determine which language most students take? Explain.

When two events are *not* mutually exclusive, you need to subtract the probability of the common outcomes to find $P(A \text{ or } B)$.

Problem 5 Finding Probability

Multiple Choice **Suppose you reach into the dish and select a token at random. What is the probability that the token is round or green?**

Think
Are the events mutually exclusive?
No; it is possible to have a round *and* green token.

(A) $\frac{2}{9}$ (B) $\frac{3}{9}$ (C) $\frac{6}{9}$ (D) $\frac{8}{9}$

$$\begin{aligned} &P(\text{round or green}) \\ &= P(\text{round}) + P(\text{green}) - P(\text{round and green}) \\ &= \frac{5}{9} + \frac{3}{9} - \frac{2}{9} = \frac{6}{9} \end{aligned}$$

The probability of selecting a round or green token is $\frac{6}{9}$, or $\frac{2}{3}$. The correct answer is C.

Got It? **5.** Suppose you select a token at random from the dish above. What is each probability?

a. the token is square or red **b.** the token is green or square

Lesson Check

Do you know HOW?

A and B are independent events. Find $P(A \text{ and } B)$.

1. $P(A) = \frac{1}{6}, P(B) = \frac{2}{5}$

2. $P(A) = \frac{9}{20}, P(B) = \frac{3}{4}$

C and D are mutually exclusive events. Find $P(C \text{ or } D)$.

3. $P(C) = \frac{2}{5}, P(D) = \frac{3}{5}$

4. $P(C) = \frac{1}{2}, P(D) = \frac{3}{8}$

5. Events A and B are not mutually exclusive. If $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{4}$, and $P(A \text{ and } B) = \frac{1}{8}$, find $P(A \text{ or } B)$.

Do you UNDERSTAND? MATHEMATICAL PRACTICES

6. **Vocabulary** Explain the difference between independent events and mutually exclusive events.

7. **Error Analysis** The weather forecast for the weekend is a 30% chance of rain on Saturday and a 70% chance of rain on Sunday. Your friend says that means there is a 100% chance of rain this weekend. What error did your friend make?

8. **Open-Ended** Describe two events that are mutually exclusive.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Tell whether the outcomes of each trial are dependent events or independent events. See Problem 1.

9. A month is selected at random; a number from 1 to 30 is selected at random.

10. A month is selected at random; a day of that month is selected at random.

11. A letter of the alphabet is selected at random; one of the remaining letters is selected at random.

12. The color of a car is selected at random; the type of transmission is selected at random.

Q and R are independent events. Find $P(Q \text{ and } R)$. See Problem 2.

13. $P(Q) = \frac{1}{4}, P(R) = \frac{2}{3}$

14. $P(Q) = \frac{12}{17}, P(R) = \frac{3}{8}$

15. $P(Q) = 0.6, P(R) = 0.9$

16. $P(Q) = \frac{1}{3}, P(R) = \frac{6}{7}$

17. **Reading** Suppose you have five books in your book bag. Three are novels, one is a biography, and one is a poetry book. Today you grab one book out of your bag without looking, and return it later. Tomorrow you do the same thing. What is the probability that you grab a novel both days?

Two fair number cubes are rolled. State whether the events are mutually exclusive. Explain your reasoning. See Problem 3.

18. The sum is a prime number; the sum is less than 4.

19. The numbers are equal; the sum is odd.

20. The product is greater than 20; the product is a multiple of 3.

S and T are mutually exclusive events. Find $P(S \text{ or } T)$. See Problem 4.

21. $P(S) = \frac{5}{8}, P(T) = \frac{1}{8}$ **22.** $P(S) = \frac{3}{5}, P(T) = \frac{1}{3}$ **23.** $P(S) = 12\%, P(T) = 27\%$

24. Population About 30% of the U.S. population is under 20 years old. About 17% of the population is over 60. What is the probability that a person chosen at random is under 20 or over 60?

A standard number cube is tossed. Find each probability. See Problem 5.

25. $P(3 \text{ or odd})$ **26.** $P(4 \text{ or even})$ **27.** $P(\text{even or less than } 4)$

28. $P(\text{odd or greater than } 2)$ **29.** $P(\text{odd or prime})$ **30.** $P(4 \text{ or less than } 6)$

B Apply 31. Suppose a number from 1 to 100 is selected at random. What is the probability that a multiple of 4 or 5 is chosen?

C 32. Think About a Plan A multiple-choice test has four choices for each answer. Suppose you make a random guess on three of the ten test questions. What is the probability that you will answer all three correctly?

- Is each guess a dependent event or an independent event?
- What is the probability that a random guess on one question will yield the correct answer?

Statistics The graph at the right shows the types of jobs held by people in the U.S. Find each probability.

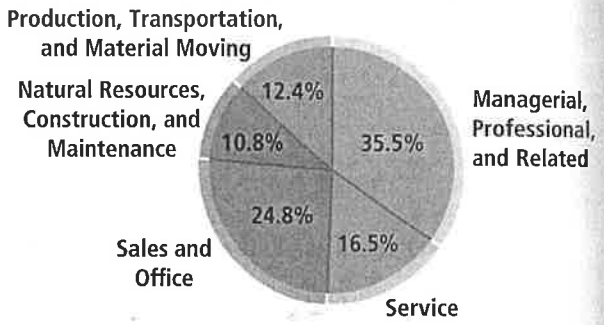
33. A person is in a service occupation.

34. A person is in service or sales and office.

35. A person is not in production, transportation, and material moving.

36. A person is neither in service nor in sales and office.

U.S. Employment, by Occupation



SOURCE: U.S. Census Bureau

A jar contains four blue marbles and two red marbles. Suppose you choose a marble at random, and do not replace it. Then you choose a second marble. Find the probability of each event.

37. You select a blue marble and then a red marble.

38. You select a red marble and then a blue marble.

39. One of the marbles you select is blue and the other is red.

40. Both of the marbles you select are red.

For each set of probabilities, determine if the events A and B are mutually exclusive.

41. $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(A \text{ or } B) = \frac{2}{3}$

42. $P(A) = \frac{1}{6}, P(B) = \frac{3}{8}, P(A \text{ or } B) = \frac{13}{24}$

Challenge

43. Two standard number cubes are rolled. What is the probability that the sum is greater than 9 or less than 6?

44. Reasoning Tatyana has $x + 2$ pens in the pocket of her backpack. Samuel has $2x - 1$ pens in the pocket of his backpack.

a. Tatyana has 2 blue pens. Find the probability that she pulls out a blue pen at random.

b. Samuel has $x - 3$ blue pens. Find the probability that he pulls out a blue pen at random.

c. Find the probability that either Tatyana or Samuel pulls out a blue pen at random.

Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

45. A bag contains 5 red marbles, 1 blue marble, 3 yellow marbles, and 2 green marbles. One marble is drawn from the bag. What is the probability that the marble is red or yellow?

46. What is the theoretical probability of getting a 1 or 6 when rolling a standard number cube?

47. The first term of an arithmetic series is 123. The common difference is 12 and the sum 1320. How many terms are in the series?

48. What is the slope of the graph of the equation $6x - 18y = -24$?

49. What is the radius of the circle with equation $x^2 - 4x + y^2 - 21 = 0$?

50. How many five-letter permutations can you form from the letters of the word COMPUTER?

Mixed Review

Find the theoretical probability of each event when rolling a standard number cube. See Lesson 11-2.

51. $P(5)$ **52.** P(an even number) **53.** P(less than 4)

Solve each equation. Check each solution. See Lesson 8-6.

54. $\frac{1}{2} - x = \frac{x}{6}$ **55.** $\frac{2}{2x - 1} = \frac{x}{3}$ **56.** $\frac{3}{2x} - \frac{2}{3x} = 5$

Solve each equation. Check your answers. See Lesson 7-6.

57. $\ln 2x = 3$ **58.** $\ln x + \ln 2 = 6$ **59.** $\ln x^2 + 1 = 5$

Get Ready! To prepare for Lesson 11-4, do Exercises 60–62.

A spinner has four equal sections that are red, blue, green, and yellow. Find each probability for two spins. See Lesson 11-3.

60. P(blue, then blue) **61.** P(red, then yellow) **62.** P(not yellow, then green)

Concept Byte

For Use With Lesson 11-3

Probability Distributions

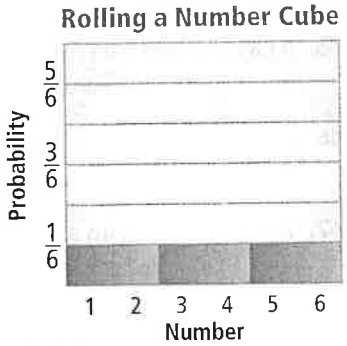
Content Standard

S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

A **probability distribution** is a function that gives the probability of each outcome in a sample space. You can use a frequency table or a graph to show a probability distribution.

The theoretical probability of rolling each number on a standard number cube is the same: $\frac{1}{6}$. It is a **uniform distribution**, a probability distribution that is equal for each event in the sample space. Here is a table and graph of its probability distribution.

Event: Roll	1	2	3	4	5	6
Frequency	1	1	1	1	1	1
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$



Now, suppose you roll two standard number cubes. You can show the probability distribution for the sum of the numbers by making a frequency table and drawing a graph.

Activity 1

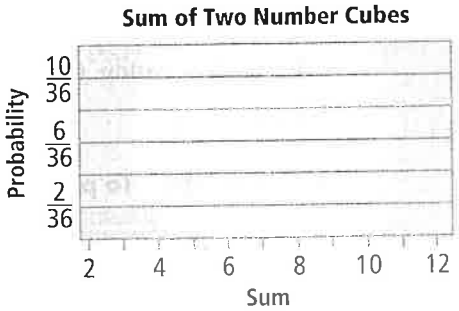
Roll a pair of standard number cubes 36 times. Record the sum for each roll.

1. Copy the frequency table below. Use your data to complete your table.

Event: Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency											
Probability											

2. Copy and complete the graph at the right using your data.

3. Make a graph of the probability distribution for the sums of two number cubes rolled 36 times, based on the *theoretical probabilities* of each sum.



4. a. Reasoning Compare the graphs. Do you think the number cubes you rolled are fair? Explain.

b. Explain why there are differences, if any, between the theoretical model and the experimental model.

When you can assign numerical values to events, the **cumulative frequency** is the number of times events with values that are less than or equal to a given value occurs. **Cumulative probability** is the probability of events occuring with values that are less than or equal to a given value.

You can use the data you collected in Activity 1 to construct a cumulative probability distribution.

Activity 2

5. Copy and complete the table below. Add the probabilities within each range to find the cumulative probalities.

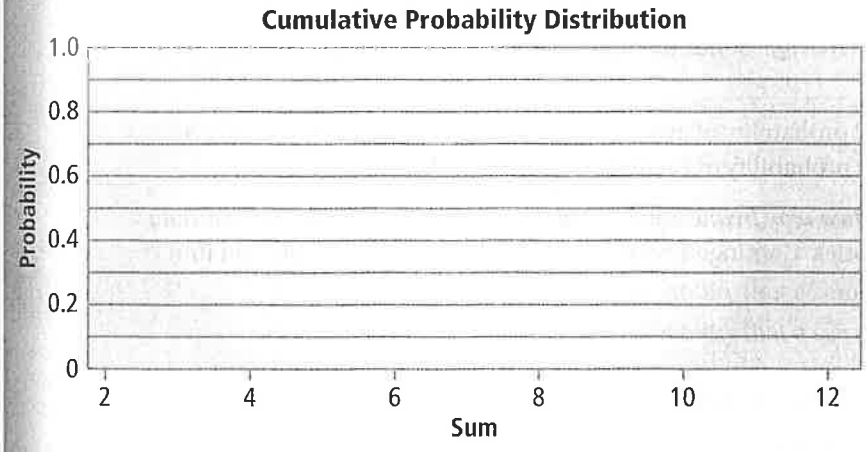
Sum	2 to 4	2 to 6	2 to 8	2 to 10	2 to 12
Cumulative Probability					

Add the probabilities for the sums of 2, 3, and 4.

Add the probabilities for sums of 5 and 6 to the previous total.

6. **Reasoning** Explain why the cumulative probability in the last interval is 1.

7. Copy the graph below and complete it using the cumulative probabilites you computed.



Exercises

8. **a.** If you roll a pair of number cubes to model a situation and observe a sum of 7 four times in a row, would you question the model? Explain.
   b. If you observed a sum of 2 four times in a row, would you question the model? Explain.

9. Use a table and a graph to show the probability distribution for the spinner {red, green, blue, yellow}.