# 11-2 Probability

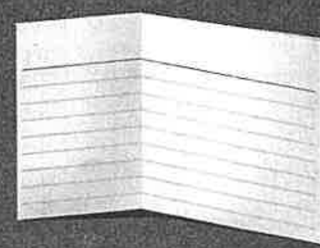
**Content Standard**

**Prepares for S.IC.2** Decide if a specified model is consistent with results from a given data generating process, e.g., using simulation.

**Objective** To find the probability of an event using theoretical, experimental, and simulation methods


**SOLVE IT! Getting Ready!**

In a probability experiment, you fold an index card slightly off center, as shown at the right. Then you drop the card from a height of several feet. What outcomes are possible? Which do you think is most likely to occur? Explain your reasoning.



**MATHEMATICAL PRACTICES** Probability measures how likely it is for an event to occur.

**Lesson Vocabulary**
- experimental probability
- simulation
- sample space
- equally likely outcomes
- theoretical probability

**Essential Understanding** The probability of an impossible event is 0 (or 0%). The probability of a certain event is 1 (or 100%). Otherwise, the probability of an event is a number between 0 and 1 (or a percent between 0% and 100%).

When you gather data from observations, you can calculate an *experimental probability.* Each observation is an experiment or a trial.

**take note** **Key Concept** **Experimental Probability**

**experimental probability** of event: $P(\text{event}) = \dfrac{\text{number of times the event occurs}}{\text{number of trials}}$

## Problem 1 Finding Experimental Probability

**Think**

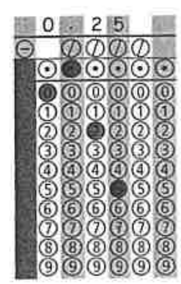
**What is a trial? What is an event?**

A trial is selecting a vehicle parking in the lot. An event is the vehicle you select being a truck.

**Gridded Response** **Of the 60 vehicles in a teachers' parking lot today, 15 are pickup trucks. What is the experimental probability that a vehicle in the lot is a pickup truck?**

$$P(\text{pickup truck}) = \frac{\text{number of pickup trucks}}{\text{number of vehicles}} = \frac{15}{60} = 0.25, \text{ or } 25\%$$

The probability that a vehicle in the lot is a pickup truck is 0.25.




**Got It? 1.** A softball player got a hit in 20 of her last 50 times at bat. What is the experimental probability that she will get a hit in her next at bat?

**Dynamic Activity** Geometric Probability

Sometimes actual trials are difficult or unreasonable to conduct. In these situations, you can estimate the experimental probability of an event by using a simulation. A **simulation** is a model of the event.

## Problem 2 Using a Simulation

**Plan**

**How do you simulate guessing one out of four?**
You can pick at random from four numbers, specifying that one of them will be the "correct" answer.

**Testing On a multiple-choice test, each item has 4 choices, but only one choice is correct. How can you simulate guessing the answers? What is the probability that you will pass the test by guessing at least 6 of 10 answers correctly?**

| Think | Write |
|---|---|
| Randomly generate 1, 2, 3, or 4 ten times. Let 1 represent a correct guess. Then 2, 3, and 4 represent incorrect guesses. | Enter **RANDINT** (1, 4, 10) on a graphing calculator. Press enter to get 10 outcomes. |
| The 10 outcomes represent 10 guesses on one test. Take the test 20 times. | 3431421212 — Three 1's. You scored 30%. Take the test 19 more times. |
| Look for tests that show 1 for at least 60% of the answers. | see list below |
| Use the probability formula. | $P(\text{passing}) = \frac{\text{number of passing tests}}{\text{number of tests taken}} = \frac{1}{20} = 0.05 = 5\%$ |

| | |
|---|---|
| 3431421212 | 3242431421 |
| 4114113144 | 3431433434 |
| 4412432243 | 4141441132 |
| 2131241131 | 2143224113 |
| 1234312221 | **2111311214** |
| 4433323314 | 3242243214 |
| 4314424244 | 3322213323 |
| 2232224422 | 3233222413 |
| 4112411124 | 2442122233 |
| 3314222333 | 3223334334 |

Only one test has at least 6 correct answers. The simulation shows

$$P(\text{passing}) = \frac{\text{number of passing tests}}{\text{number of tests taken}} = \frac{1}{20} = 0.05 = 5\%$$

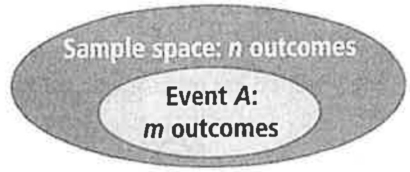
**Got It? 2.** In Problem 2, what is the probability of passing if a passing score is 50% or better?

The set of all possible outcomes to an experiment or activity is a **sample space**. When each outcome in a sample space has the same chance of occurring, the outcomes are **equally likely outcomes**.

For one roll of a standard number cube, there are six equally likely outcomes in the sample space. You can calculate *theoretical probability* as a ratio of outcomes.

## Key Concept Theoretical Probability

If a sample space has $n$ equally likely outcomes and an event $A$ occurs in $m$ of these outcomes, then the **theoretical probability** of event $A$ is $P(A) = \frac{m}{n}$.




## Problem 3 Finding Theoretical Probability

**What is the theoretical probability of each event?**

**A** **getting a 5 on one roll of a standard number cube**

There are six equally likely outcomes, 1, 2, 3, 4, 5, and 6. The number 5 occurs one way.

$$P(5) = \frac{1}{6}$$

**Plan**

**How many outcomes are there?**
Each cube has six numbers on it, so there are $6 \cdot 6 = 36$ outcomes.

**B** **getting a sum of 5 on one roll of two standard number cubes**

There are 36 possible equally likely outcomes. The favorable outcomes are those with a sum of 5.

$$P(\text{sum } 5) = \frac{4}{36} = \frac{1}{9}$$

**Got It?** **3. a.** What is the theoretical probability of getting a sum that is an odd number on one roll of two standard number cubes?

**b. Reasoning** Without calculating the probability, is it more likely to get an even or odd number on one roll of a standard number cube? Explain.

It can be easier to use *combinatorics* to find theoretical probability rather than listing and counting all the equally likely outcomes. Combinatorics include the Fundamental Counting Principle, and ways to count permutations and combinations.

## Problem 4 Finding Probability Using Combinatorics

**What is the theoretical probability of being dealt exactly two 7's in a 5-card hand from a standard 52-card deck?**

**Think**

**Should you use permutations or combinations?**
Order does not matter. Use combinations.

A standard 52-card deck has 4 suits; hearts, diamonds, clubs, and spades. Each suit contains cards numbered 2 though 10, and an ace, jack, queen, and king.

The number of combinations of two 7's from four 7's $= {}_4C_2$.

The number of combinations of 3 non-sevens from 48 other cards $= {}_{48}C_3$.

The number of 5-card hands with two 7's $= {}_4C_2 \cdot {}_{48}C_3$.

The number of possible 5-card hands $= {}_{52}C_5$.

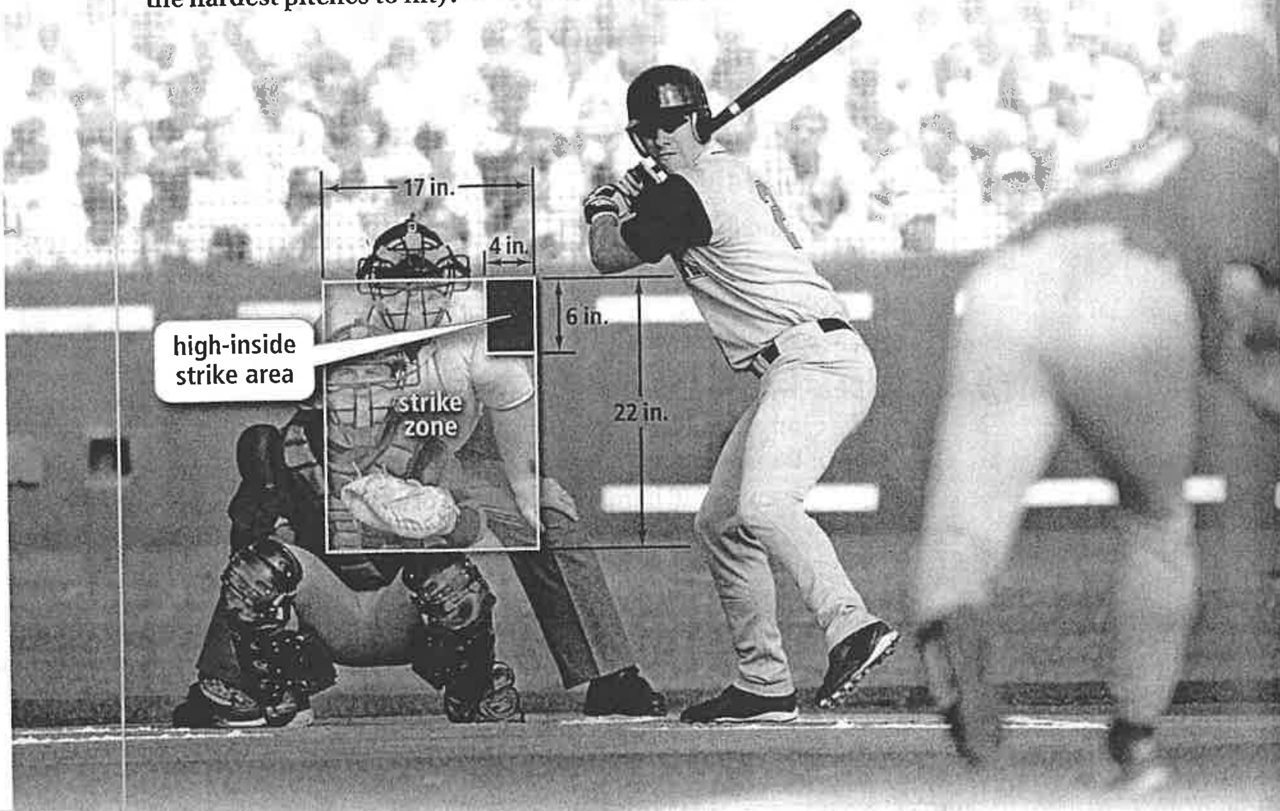
$$P(\text{hand with two 7's}) = \frac{{}_4C_2 \cdot {}_{48}C_3}{{}_{52}C_5} = \frac{103{,}776}{2{,}598{,}960} \approx 0.0399, \text{ or about } 4\%.$$

**Got It?** 4. What is the theoretical probability of being dealt all four 7's in a 5-card hand?

Sometimes you can use areas to find a theoretical probability.

## Problem 5 Finding Geometric Probability

**Geometry A batter's strike zone depends on the height and stance of the batter. What is the geometric probability that a baseball thrown at random within the batter's strike zone, as shown in the figure below, will be a high-inside strike (one of the hardest pitches to hit)?**




**Think**

**What are the favorable outcomes? All outcomes?**

Favorable outcomes are points in the high-inside region. All outcomes are points in the strike zone.

$$P(\text{high-inside strike}) = \frac{\text{area of high-inside strike zone}}{\text{area of total strike zone}}$$

$$= \frac{4 \cdot 6}{17 \cdot 22}$$

$$\approx 0.064$$

For a baseball thrown at random in the batter's strike zone, the probability that it will be a high-inside strike is about 6.4%.

**Got It?** 5. Suppose a batter's strike zone is 15 in.-by-20 in. and the high-inside strike zone is 3 in.-by-5 in. What is the probability that a baseball thrown at random within the strike zone will be a high-inside strike?

## Lesson Check

### Do you know HOW?

1. What is the experimental probability a quarterback will complete his next pass if he has completed 30 of his last 40 passes?

2. What is the experimental probability a quarterback will complete his next pass if he has completed 36 of his last 45 passes?

**Find the theoretical probability of each event when rolling a standard number cube.**

3. $P(3)$

4. $P(2 \text{ or } 4)$

### Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Vocabulary** Explain the difference between experimental probability, theoretical probability, and geometric probability.

6. **Writing** Describe three ways you could simulate answering a true-false question.

7. **Reasoning** Why is a simulation better the more times you perform it?

## Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

**A Practice**

See Problem 1.

8. A class tossed coins and recorded 161 heads and 179 tails. What is the experimental probability of heads? Of tails?

9. Another class rolled number cubes. Their results are shown in the table. What is the experimental probability of rolling each number?

| Number | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Occurrences | 42 | 44 | 45 | 44 | 47 | 46 |

See Problem 2.

**Graphing Calculator** **For Exercises 10–12, define a simulation by telling how you represent correct answers, incorrect answers, and the quiz. Use your simulation to find each experimental probability.**

10. If you guess the answers at random, what is the probability of getting at least two correct answers on a five-question true-or-false quiz?

11. If you guess the answers at random, what is the probability of getting at least three correct answers on a five-question true-or-false quiz?

12. A five-question multiple-choice quiz has five choices for each answer. What is the probability of correctly guessing at random exactly one correct answer? Exactly two correct answers? Exactly three correct answers? (*Hint:* You could let any two digits represent correct answers, and the other digits represent wrong answers.)

See Problem 3.

**A jar contains 30 red marbles, 50 blue marbles, and 20 white marbles. You pick one marble from the jar at random. Find each theoretical probability.**

13. $P(\text{red})$

14. $P(\text{blue})$

15. $P(\text{not white})$

16. $P(\text{red or blue})$

**A bag contains 36 red blocks, 48 green blocks, 22 yellow blocks, and 19 purple blocks. You pick one block from the bag at random. Find each theoretical probability.**

**17.** $P$(green) **18.** $P$(purple) **19.** $P$(not yellow)

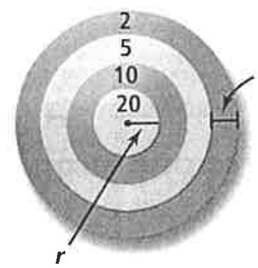
**20.** $P$(green or yellow) **21.** $P$(yellow or not green) **22.** $P$(purple or not red)

See Problem 4.

**23. Games** A group of 30 students from your school is part of the audience for a TV game show. The total number of people in the audience is 150. What is the theoretical probability of 3 students from your school being selected as contestants out of 9 possible contestant spots?

See Problem 5.

**Geometry Suppose that a dart lands at random on the dartboard shown at the right. Find each theoretical probability.**




**24.** The dart lands in the bull's-eye.

**25.** The dart lands in a green region.

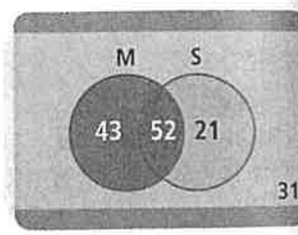
**26.** The dart scores at least 10 points.

**27.** The dart scores less than 10 points.

**B Apply**

**28. Think About a Plan** Suppose you roll two standard number cubes. What is the theoretical probability of getting a sum of 7?
- What is the sample space?
- How many outcomes are there?

**In a class of 147 students, 95 are taking math (M), 73 are taking science (S), and 52 are taking both math and science. One student is picked at random. Find each probability.**




**29.** $P$(taking math or science or both)

**30.** $P$(not taking math)

**31.** $P$(taking math but not science)

**32.** $P$(taking neither math nor science)

**33. Lottery** A lottery has 53 numbers from which five are drawn at random. Each number can only be drawn once. What is the probability of your lottery ticket matching all five numbers in any order?

**34. a. Sports** Out of four games, team A has won one game and team B has won three games in a championship series. What is the experimental probability that team A wins the next game? That team B wins the next game?

**b. Reasoning** Do you think that experimental probability is a good predictor of the winner of the next game? Explain.

**35. Writing** Explain what you would need to know to determine the theoretical probability of a five-digit postal ZIP code ending in 1.

**Challenge**

36. Assume that an event is neither certain nor impossible. Then the odds in favor of the event are the ratio of the number of favorable outcomes to the number of unfavorable outcomes.
    a. If the odds in favor of the event are $a$ to $b$, or $\frac{a}{b}$, what is the probability of the event?
    b. If the probability of the event is $\frac{a}{b}$, what are the odds in favor of the event?
    c. Would you rather play a game in which your odds of winning are $\frac{1}{2}$, or a game in which your probability of winning is $\frac{1}{2}$? Explain.

## Standardized Test Prep

SAT/ACT

37. What is the theoretical probability of getting a 2 or a 3 when rolling a standard number cube?

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$

38. Which expression is equivalent to $\left(n^{\frac{3}{2}} \div n^{-\frac{1}{6}}\right)^{-3}$?

(F) $n^{27}$ (G) $n^{-27}$ (H) $n^{-4}$ (I) $n^{-5}$

39. How can you rewrite the equation $x^2 + 12x + 5 = 3$ so the left side of the equation is in the form $(x + a)^2$?

(A) $(x - 6)^2 = 28$
(B) $(x + 6)^2 = 34$
(C) $(x + 6)^2 = 39$
(D) $(x + 12)^2 = -2$

40. How many ways are there to select 25 books from a collection of 27 books?

(F) 702 (G) $5.4 \times 1027$ (H) 351 (I) 675

Short Response

41. Use the center and radius to graph the circle with equation $(x + 4)^2 + (y - 2)^2 = 16$.

## Mixed Review

**Evaluate each expression.** **See Lesson 11-1.**

42. $_5P_2$ 43. $_7P_4$ 44. $_5C_3$ 45. $_{10}C_8$

**Add or subtract. Simplify where possible.** **See Lesson 8-5.**

46. $\frac{5}{a^2b} - \frac{7a}{5b^2}$ 47. $\frac{3}{p} + \frac{7}{q}$ 48. $\frac{x}{x-5} + \frac{x}{5-x}$

### Get Ready! To prepare for Lesson 11-3, do Exercises 49–51.

**A bag contains 24 green marbles, 22 blue marbles, 14 yellow marbles, and 12 red marbles. Suppose you pick one marble at random. What is each probability?** **See Lesson 11-2.**

49. $P$(yellow) 50. $P$(not blue) 51. $P$(green or red)